Gyrokinetics at Imperial College

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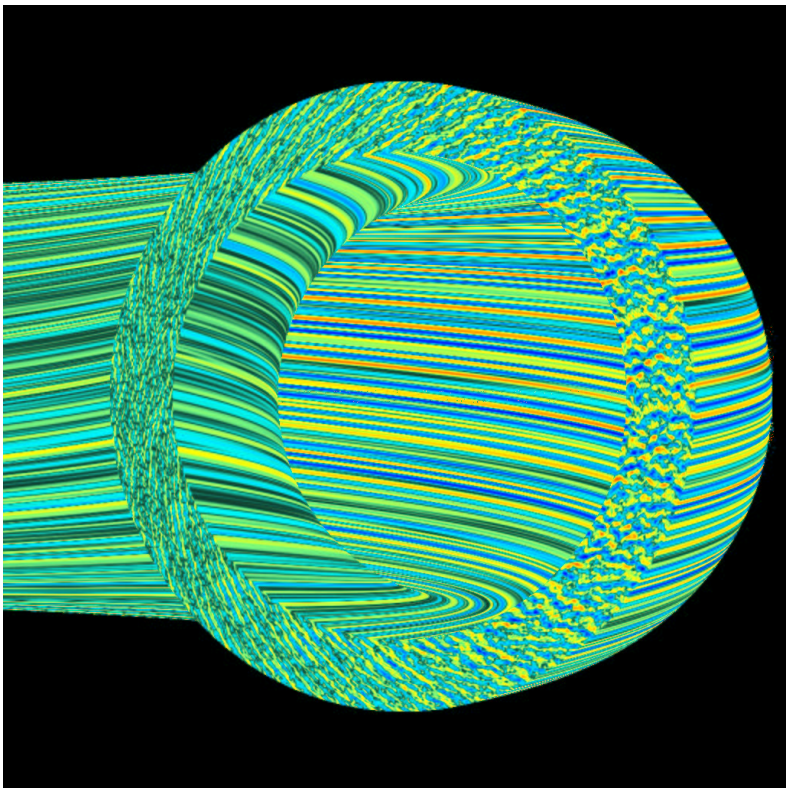


Overview

1. Gyrokinetic equations
2. General geometry, simulation domain
3. Microinstability basics: linear
4. ETG turbulence
5. Zonal flow dynamics
6. Dimits shift (nonlinear instability threshold)
7. Dominant role of collisions
8. Opportunities

Gyrokinetic Equation

- Gyrokinetic equation appropriate if

$$\frac{\omega}{\Omega} \sim \frac{k_\parallel}{k_\perp} \sim \frac{\delta f}{f} \sim \frac{e\delta\Phi}{T} \sim \frac{\delta B}{B} \sim \frac{\rho}{L} \ll 1$$

- Gyrokinetic equation describes evolution of perturbed distribution function $h = h_s(\rho, \alpha, \theta, \epsilon, \mu; t)$. For $F_0 = F_0(\epsilon, \Psi)$:

$$\left(\frac{d}{dt} + v_\parallel \hat{\mathbf{b}} \cdot \nabla + i\omega_d + C\right) h = i\omega_*^T \chi - q\frac{\partial F_0}{\partial \epsilon}\frac{\partial \chi}{\partial t}.$$

- The total derivative is

$$\frac{dh}{dt} = \frac{\partial h}{\partial t} + \frac{c}{B}\{\chi, h\}.$$

- The drift frequency $i\omega_*^T = n_0 c\, \partial F_0/\partial\Psi$, where n_0 is the toroidal mode number of the perturbation and Ψ is the equilibrium poloidal magnetic flux enclosed by the magnetic surface of interest.

- The perpendicular drifts (curvature, grad-B) are

$$\omega_d = \mathbf{k}_\perp \cdot \mathbf{B}_0 \times \left(m v_\parallel^2 \hat{\mathbf{b}} \cdot \nabla \hat{\mathbf{b}} + \mu \nabla B_0\right) / (m B_0 \Omega),$$

- The fields are represented by

$$\chi = J_0(\gamma)\left(\Phi - \frac{v_\parallel}{c} A_\parallel\right) + \frac{J_1(\gamma)}{\gamma}\frac{m v_\perp^2}{q}\frac{\delta B_\parallel}{B}.$$

Here, $\gamma = k_\perp v_\perp / \Omega$.

Gyrokinetic Maxwell Equations

- Fields determined by Maxwell equations, neglecting displacement current.
- Poisson's equation:

$$\nabla_\perp^2 \Phi = 4\pi \sum_s \int d^3v\, q \left[q\Phi \frac{\partial F_0}{\partial \epsilon} + h \exp\left(iL\right) \right],$$

  where $L = (\mathbf{v} \times \hat{\mathbf{b}} \cdot k_\perp)/\Omega$ accounts for the gyrophase dependence.
- Preferred velocity space coordinates are (ϵ, μ, ξ), so that

$$\int d^3v = \frac{B}{m^2} \int \frac{d\epsilon\, d\mu\, d\xi}{|v_\parallel|} \equiv \frac{1}{2\pi} \int d^2v\, d\xi$$

- Integrate over the gyrophase to find

$$\nabla_\perp^2 \Phi = 4\pi \sum_s \int d^2v\, q \left[q\Phi \frac{\partial F_0}{\partial \epsilon} + J_0(\gamma) h \right]$$

- Similarly, Ampere's equation provides the two components of the perturbed magnetic field:

$$\nabla_\perp^2 A_\parallel = -\frac{4\pi}{c} \sum_s \int d^2v\, q v_\parallel J_0(\gamma) h$$

$$\frac{\delta B_\parallel}{B} = -\frac{4\pi}{B^2} \sum_s \int d^2v\, m v_\perp^2 \frac{J_1(\gamma)}{\gamma} h$$

General Geometry

- Metric coefficients for general geometry calculations are well known. Given a Clebsch representation of the magnetic field:

$$\mathbf{B} = \nabla\alpha \times \nabla\psi$$

  one can calculate all needed quantities.

- In the field-line-following (ballooning) limit, perturbed quantities are represented by

$$A = \hat{A}(\theta) \exp(iS)$$

  where $\hat{\mathbf{b}} \cdot \nabla S = 0$. This takes into account the fact that the perturbations tend to be slowly varying along the field line, and allows for rapid variation across the field line.

- To make contact with the ballooning approximation and with field-line following coordinates, choose

$$S = n_0 (\alpha + q\theta_0),$$

  where n_0 is an integer, θ_0 is the ballooning parameter, $q(\Psi) = d\Psi_T/d\Psi$, $\Psi_T = (2\pi)^{-2} \int_V d\tau \mathbf{B} \cdot \nabla\phi$ is the toroidal flux, and $d\tau$ is the volume element.

- Simulation coordinates are therefore (Ψ, α, θ).

Flux-Tube Simulation Domain

We use field-line-following coordinates in a "flux-tube."

- Simulation domain has a "small" perpendicular cross-section:
  The simulation domain should be at least a few turbulent correlation lengths long in each direction.
- The variation of the gradient scale lengths across the simulation domain is assumed to be small.
- The energy, momentum and/or particle sources and sinks inside the flux-tube are assumed to be small.
- Radial periodicity prevents artificial flattening of driving gradients.
- Non-trivial periodicity condition along the field line makes field-line-following coordinates exactly equivalent to ballooning transformation when parallel correlation length

$$\lambda_{\parallel} < 2\pi q R.$$

- For small ρ/a, flux tube assumptions reduce the total simulation volume significantly.

Summary of Numerical Issues: GS2

- Electrostatic and electromagnetic perturbations treated on equal footing.
- 5-D distribution function $h = h_s(\Psi, \alpha, \theta, \epsilon, \mu; t)$ evolved in time for each species.
- Implicit integration of fast motion along field lines reduces time step restrictions and automatically recovers appropriate orbit averages, without any subsidiary ordering of nonlinear terms.
- Velocity-space filamentation not a problem for realistic collision frequencies with full Lorentz collision operator.
- GS2 uses spectral decomposition in the two directions perpendicular to the field line and for energy and pitch angle for good accuracy with minimal memory & CPU usage.
- Pseudo-spectral evaluation of nonlinear terms is fast, without introducing numerical dissipation.
- Flux-tube simulation domain is maximally consistent with GK ordering.
- Source code available on-line; presently 18 users.

Microinstability Basics: Linear

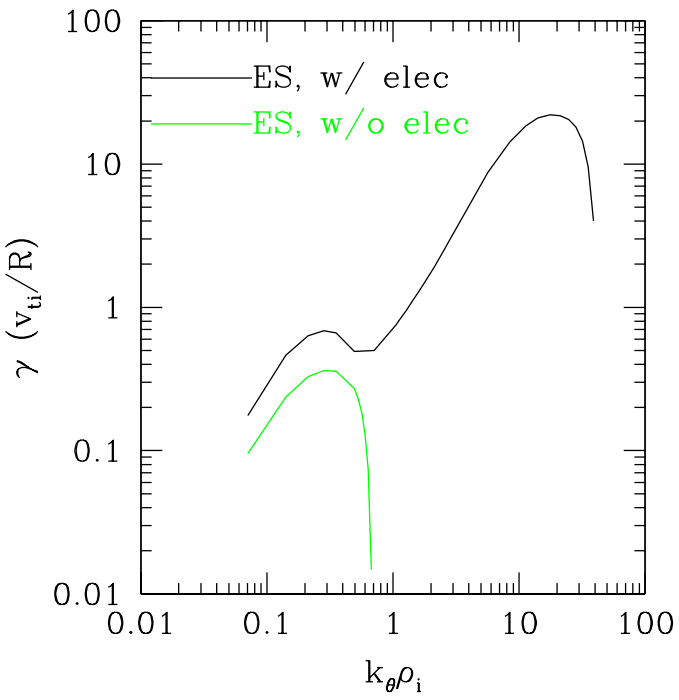
- Instabilities driven by gradients in temperature, momentum and/or density.
- Growth rates typically proportional to ω_*, wavelengths typically several gyro-radii.
- In core plasma, three most important instabilities are:
  1. ITG, driven by Ion Temperature Gradient;
  2. TEM (Trapped Electron Mode) driven mainly by density gradients;
  3. ETG, driven by Electron Temperature Gradient;
- Linear properties well-understood theoretically, but realistic assessment of stability requires state-of-the-art calculation since typically $k_\perp \rho \sim 1$,

$$\omega \sim k_\parallel v_t, \quad \omega_d, \quad \omega_b$$

  multiple gradients co-exist, and geometric effects (magnetic shear, shaping, *etc.*) are strong.

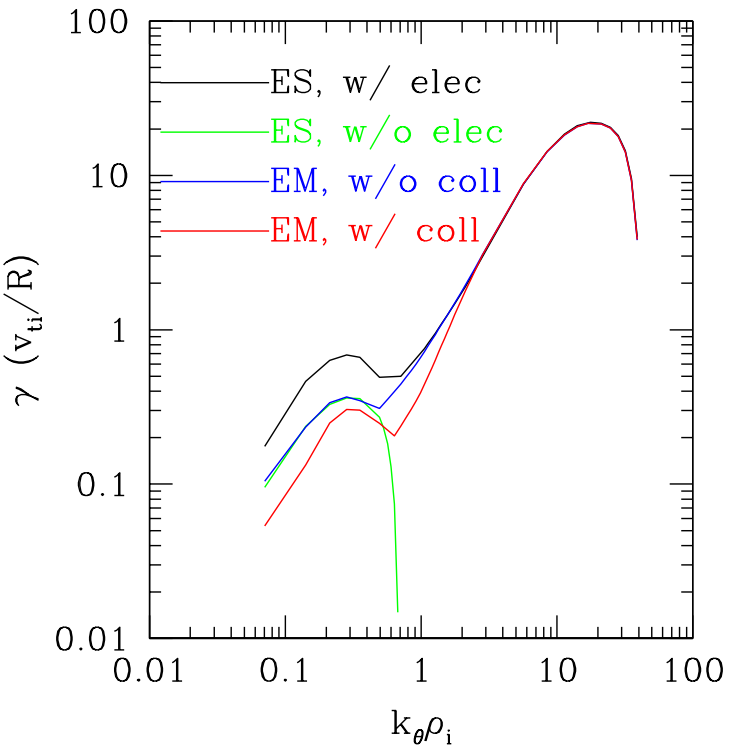
Electron Dynamics Enhances Instability

- Early simulations assumed electron response was adiabatic. Allowed reasonable assessment of ITG-induced energy transport.
- To understand particle and electron energy transport, electron dynamics must be included.
- Trapped electrons enhance long wavelength instabilities; influence reduced by collisions. Passing electrons enable ETG modes.



Electron Dynamics Introduces Electromagnetic Turbulence

- Passing electrons carry perturbed currents, which may introduce small scale magnetic islands, but which also modestly stabilize ITG modes.
- Possibility of magnetic electron energy transport exists.
- Toroidal ETG modes linearly unaffected by finite β.



ETG Modes Often Ignored Because Mixing Length Transport is Small

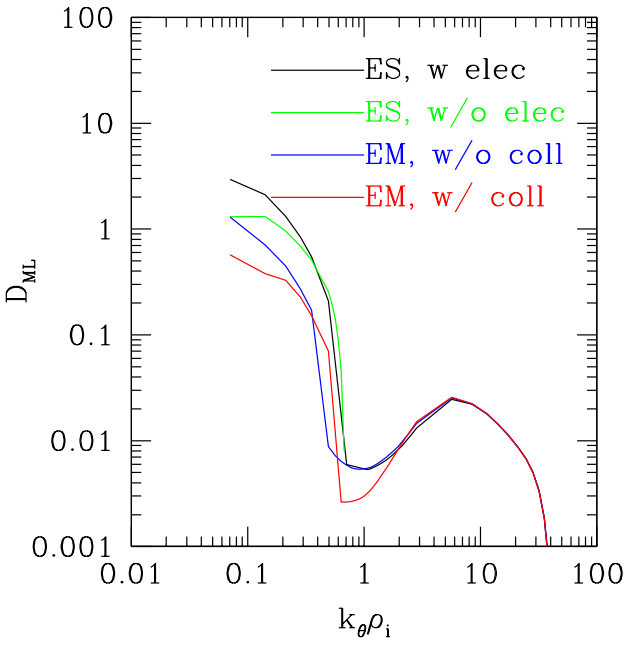
- Mixing length estimate of transport is

$$D_{\mathrm{ML}} \sim \frac{\gamma}{\bar{k}_\perp^2}$$

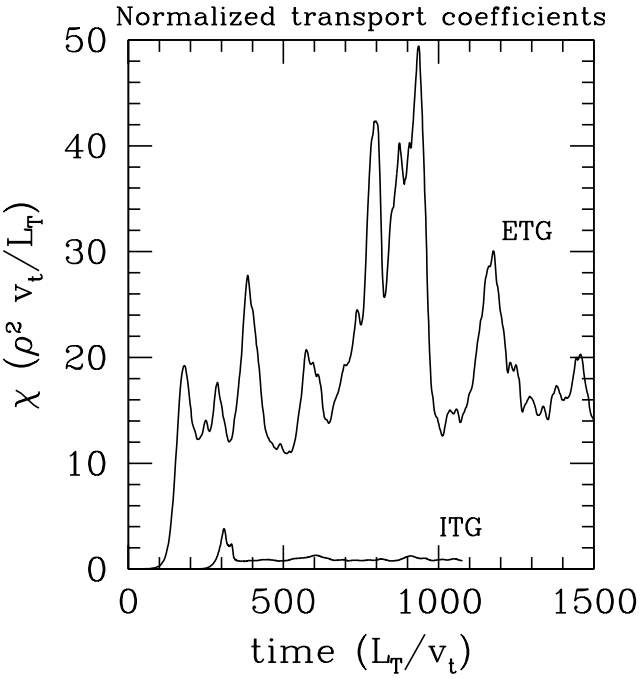
where γ is the linear growth rate. Results from balance of linear and nonlinear terms:

$$\frac{\partial h}{\partial t} + \frac{c}{B}\{\chi, h\} + v_\parallel \hat{\mathbf{b}} \cdot \nabla h + i\omega_d h + Ch = i\omega_*^T \chi - q\frac{\partial F_0}{\partial \epsilon}\frac{\partial \chi}{\partial t}$$

- Because $\gamma \sim \omega_*$, this decreases at high $k_\perp$.



Nonlinear ETG Simulations Show Transport Larger than Mixing Length



- ETG and ITG mixing length expectation:

$$\frac{\chi_e^{\mathrm{ETG}}}{\rho_e^2 v_{te}/L_{Te}} \sim \frac{\chi_i^{\mathrm{ITG}}}{\rho_i^2 v_{ti}/L_{Ti}}$$

  so that $\chi_e^{\mathrm{ETG}}/\chi_i^{\mathrm{ITG}} \sim \sqrt{m_e/m_i} \sim 1/60$

- EM GK simulations show that "streamers" can lead to large *normalized* ETG transport coefficients, so that $\chi_e^{\mathrm{ETG}} \sim \chi_i^{\mathrm{ITG}}$

- Occurs because secondary instabilities that produce saturation are weak in the ETG limit

Radially Elongated Streamers Found in ETG Simulations

- Governing equation is

$$\left[\left(\frac{\tau}{1+\tau}\right)\gamma + k_x^2\bar{\gamma}\right]\tilde{\psi} = \partial_y\left[\bar{\gamma}^2\partial_y(\tilde{\psi}/\bar{\gamma})\right]$$

  with $\bar{\gamma} = \gamma - ik_x\psi_p'(y)$ and $\tau = Z_{\text{eff}}T_e/T_i$.

- Here, ψ_p is the guiding center electrostatic potential from ETG eigenmode.

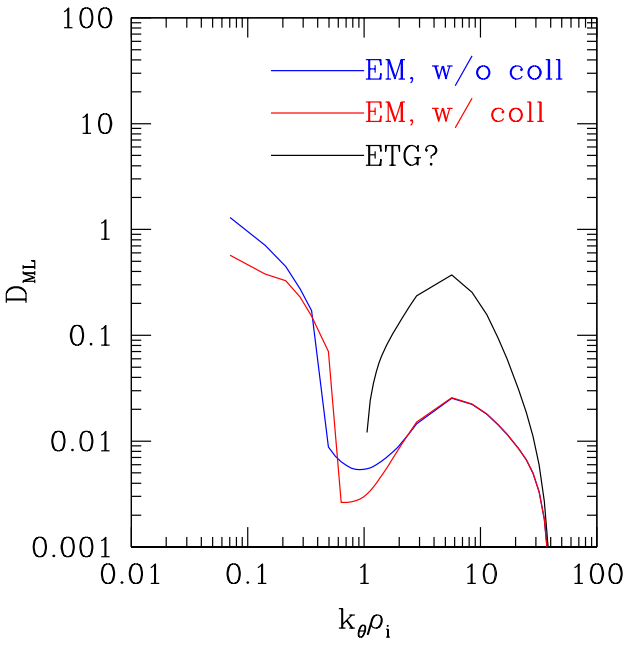
- Instability differs from conventional Kelvin-Helmholtz (KH) because of the first term on the left-hand side (the ion response).

- Balancing ion response with the other terms leads to a maximum growth rate $\sim k_p^4\psi_{p0}$ for $k_x \sim k_p$.

- This is weaker than conventional KH by a factor of $(k_\perp\rho_e)^2$, and still weaker compared to ITG secondary instabilities.

ETG Modes Likely Contribute to Electron Energy Transport

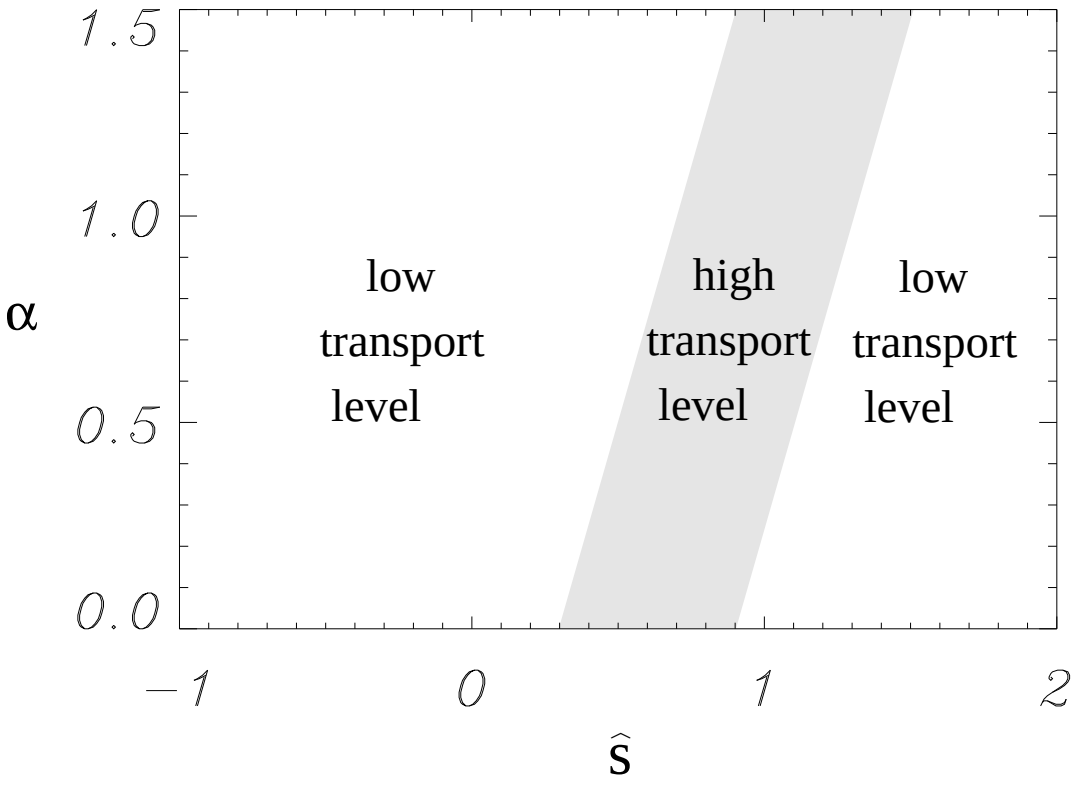
- Because $k_\perp \rho_i \gg 1$, one does not expect (or find) significant particle or ion energy transport.
- Simple expectation from secondary stability saturation mechanism:



- However, mysteries remain. Preliminary evidence suggests ETG $\chi_e \sim 1/\beta$ in some regimes. Important for Ohmic core? Pedestal? ITB?
- Very hard to stabilize ETG modes with velocity shear. More interesting *second microstability* possibilities exist.

Strongly Reversed Magnetic Shear Can Reduce ETG Transport

- For typical core gradients, approximately 25 nonlinear runs used to map out suggestive diagram:



- For steeper gradients (such as may be found in ITB's) the high transport region extends to negative magnetic shear.

- Strongly reversed magnetic shear should reduce electron energy transport in ITB. Feedback loop possible?

For Modes with $k_\perp \rho_i < 1$, Secondary Instabilities are Strong

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- Governing equation is

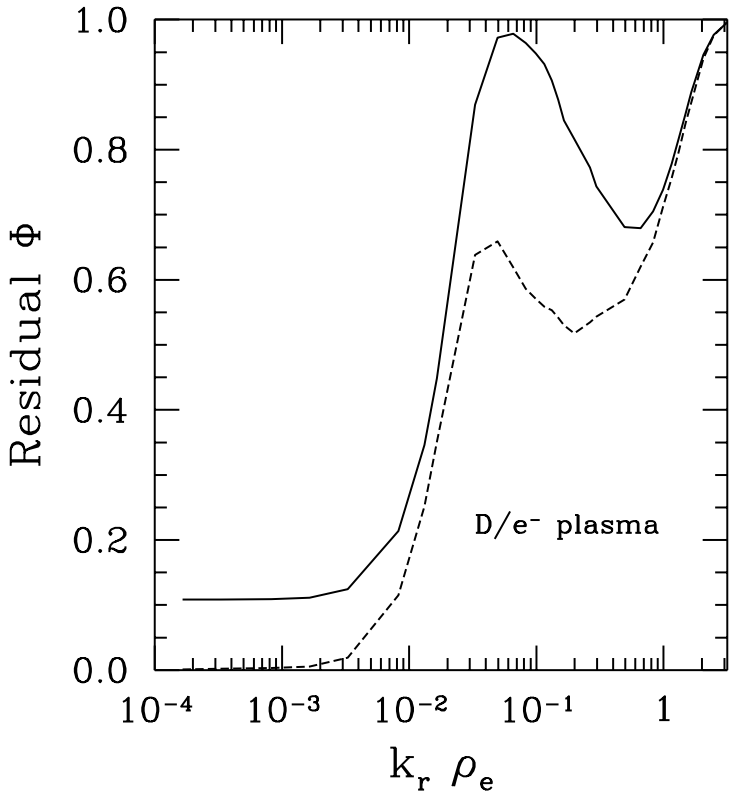
$$-(1+\tau k_x^2)\bar{\gamma}\langle\tilde{\psi}\rangle + \left[\gamma + k_x^2\Gamma\right]\tilde{\psi} = \partial_y\left[\bar{\gamma}\partial_y(\tilde{\psi}/\bar{\gamma})\right]$$

  with $\tau = T_i/T_e$, $\Gamma = \tau\gamma + \bar{\gamma}_T$ and

$$\bar{\gamma} = \gamma - ik_x\psi_l'(y), \qquad \bar{\gamma}_T = \gamma - ik_x[(1+\tau)\psi_l'(y) + \tau T_l'(y)].$$

- Here, ψ_l and T_l are the guiding center electrostatic potential and temperature from an ITG eigenmode.
- Instability differs from conventional Kelvin-Helmholtz (KH) mainly because of the first two terms on the left-hand side (the electron response).
- First term leads to enhancement of instability by a factor $1/(k_\perp\rho_i)$.
- As a result, secondary instabilities in ITG limit are strong, driving zonal flows robustly.

Rosenbluth and Hinton: Linearly Undamped Zonal Flows Exist



- Dielectric shielding of static component of driven zonal flow is incomplete. At long wavelengths, trapped ions adjust to permit linearly undamped zonal flows in collisionless limit.

- Lower (dashed) curve has $r/R = 0$. Upper (solid) curve has $r/R = 0.18$.

- Puzzle: Dielectric shielding decreases at short wavelength. Why isn't ETG transport negligible?

Tertiary Instabilities Control Zonal Flow

- Zonal flows driven strongly by ITG secondaries, weakly by ETG secondaries.
- Zonal flows themselves weakly unstable at long wavelength, strongly unstable at sub-ρ_i scales.
- At long wavelengths, governing equation is

$$\left[(1+(1+\tau)k_x^2)\bar{\gamma} + ik_y(k_y^2\tau T_0' - \tau\psi_0''')\right]\tilde{\psi}$$
$$= \partial_x\left[(\tau\gamma + \bar{\gamma}_T)\bar{\gamma}\partial_x(\tilde{\psi}/\bar{\gamma})\right]$$

  with

$$\bar{\gamma} = \gamma + ik_y\psi_0'(x), \qquad \bar{\gamma}_T = \gamma + ik_y[(1+\tau)\psi_0'(x) + \tau T_0'(x)]$$

- Here, ψ_0 and T_0 are the guiding center electrostatic potential and temperature from zonal flow.
- Natural scale of eigenmode which is localized at x_0 is

$$\lambda = (2\tau T_0'/\psi_0''')^{1/4}|_{x_0}$$

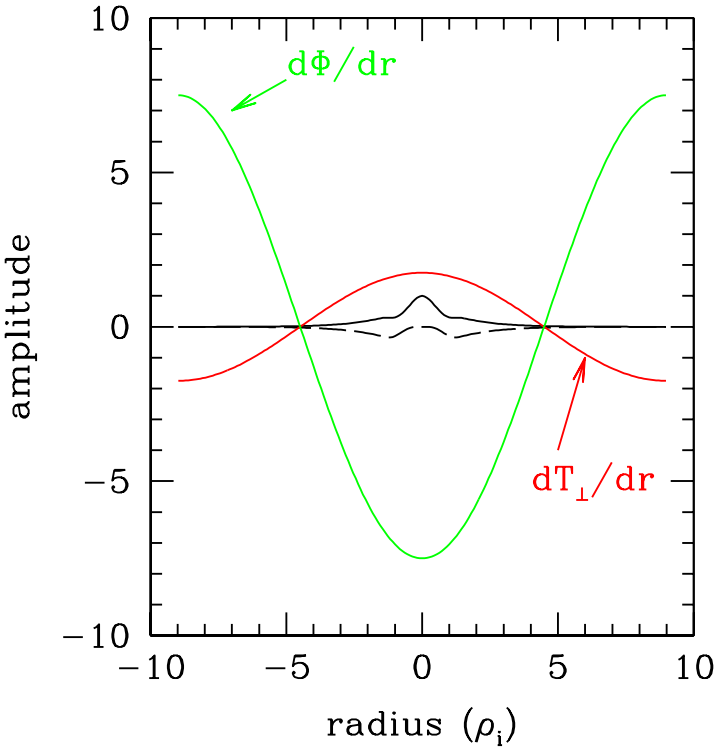
- Ignoring background gradients, maximum (over k_y) growth rate of instability is

$$\gamma \simeq 0.55 k_y\sqrt{\tau T_0'\psi_0'''/2}$$

  in good agreement with the simulations.

Tertiary Instability Driven by Combination of Gradients Inherent to RH Flows

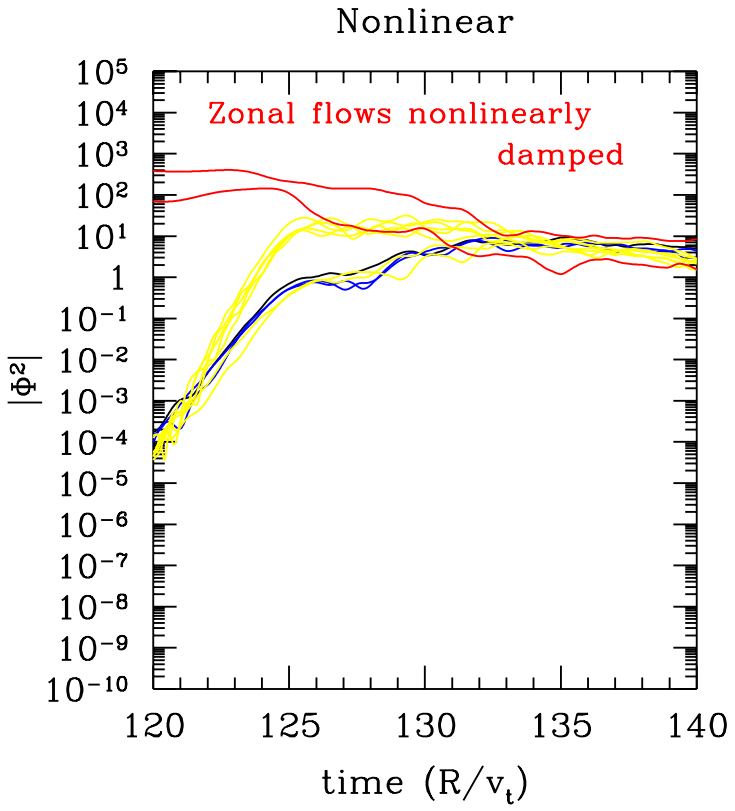
- Example calculation for typical parameters.
- $T_\perp$ and Φ perturbations easily shown to be out of phase for Rosenbluth-Hinton zonal flows.



- Precise stability threshold determined by including magnetic shear, curvature, *etc.* is complicated. In general, weak zonal flows are stable.

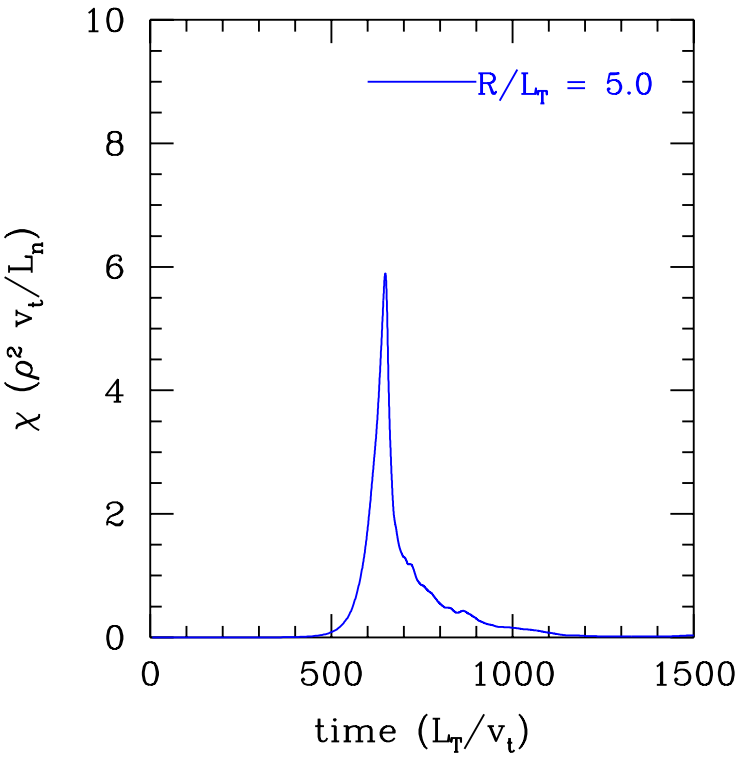
Tertiary Instability Damps Zonal Flows

- Numerical test required to determine whether this instability is strong enough to regulate zonal flows.
- Initialized zonal flows (red) at high amplitude, all other Fourier modes at low level. No equilibrium gradients. Without tertiary instability, these flows are undamped.
- Teritiary modes (yellow) grow at expected rate, observed to break up zonal flows.



At Long Wavelengths, It Is Possible to Quench Turbulence With Self-Generated Zonal Flows

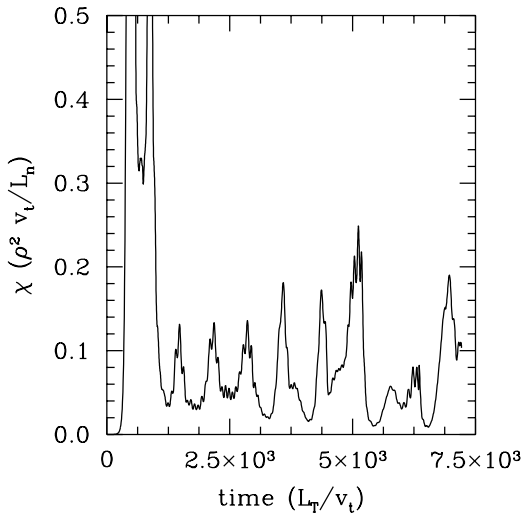
- Consider parameters for which there is linear instability.
- RH zonal flows that are stable to small perturbations may exist. For weakly unstable modes, these RH zonal flows restore linear stability.



The turbulent heat flux quickly falls to zero.

Ion-ion collisions Damp Self-Generated Zonal Flows

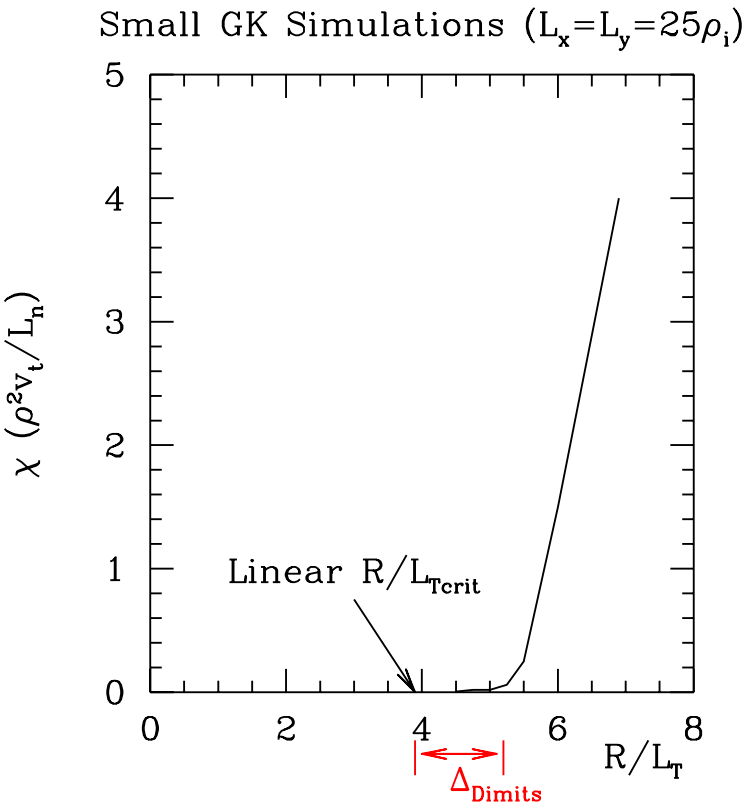
- As pointed out by Rosenbluth and Hinton, and Diamond and Rosenbluth, ion-ion collisions can in principle regulate zonal flows.
- In such a case, increasing temperature would decrease ν_{ii}, allowing the zonal flow amplitude to increase, and therefore reducing the turbulent transport. Hypothesized to be important in ITER-class tokamak.
- Z. Lin, *et al.*, confirmed the role of ion-ion collisions when self-generated RH flows would otherwise restore linear stability. Bursty behaviour observed.
- Here's an example from `GS2`:



We will return to the role of ν_{ii} later in this talk.

In Such Regimes, Linear Threshold is Basically Irrelevant

- In collisionless plasma, regime of nonlinearly-upshifted critical gradient for long-wavelength turbulence discovered by A. Dimits.
- Easily reproduced. Model case:



- Important question: What defines this regime? *I.e.*, what limits the Dimits shift?

Dimits Shift Limited by Tertiary Instability

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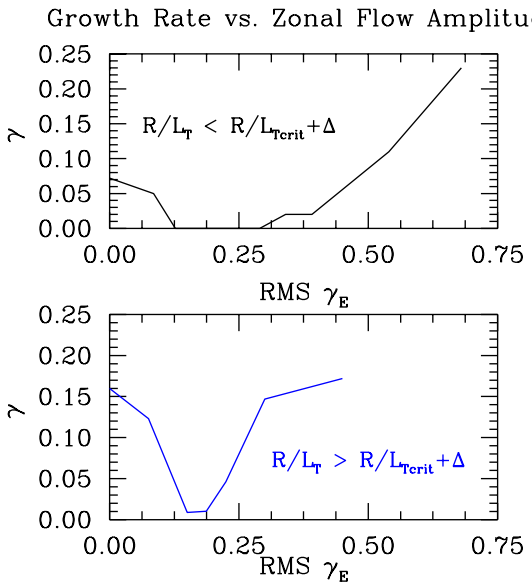
1. Consider several nonlinear simulations for which the spectrum of zonal flows is taken from a nonlinear simulation for which $R/L_T < R/L_{T\mathrm{crit}} + \Delta$ by a small amount.

2. Calculate stability of zonal flows as a function of amplitude of zonal flows. Record linear growth rate when it exists.

3. Repeat calculations, except increase background temperature gradient to be slightly greater than $R/L_{T\mathrm{crit}} + \Delta$.

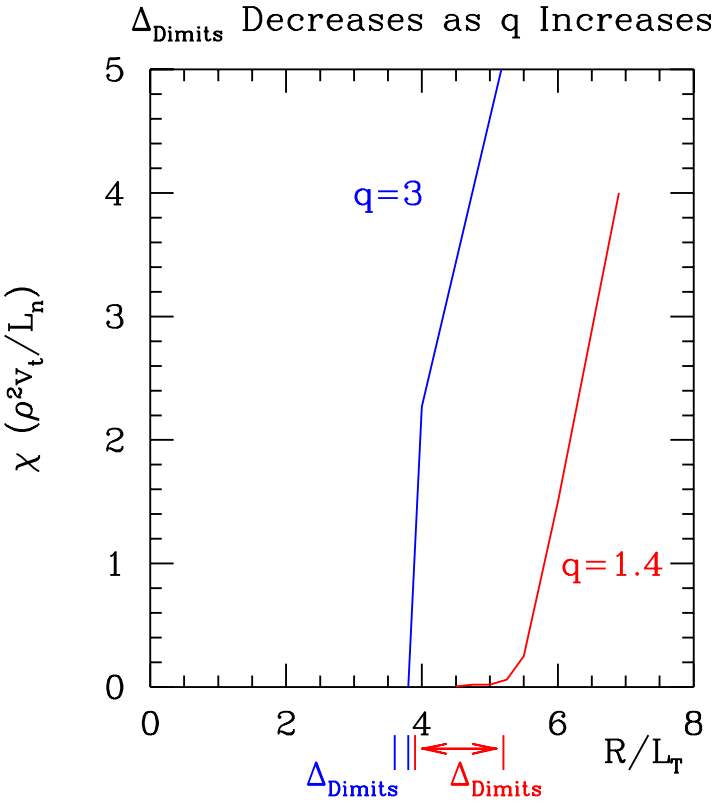


- Tertiary instability threshold corresponds to $R/L_T + \Delta$.

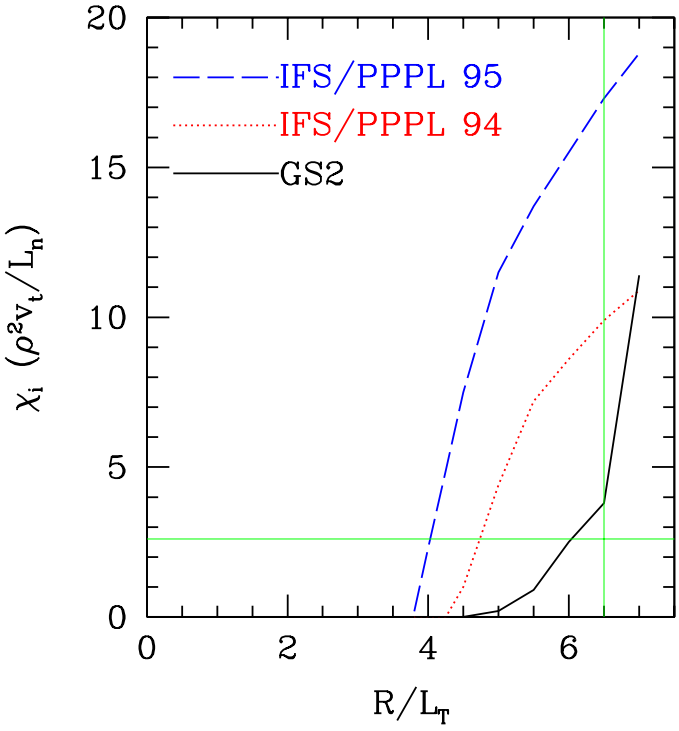
Self-generated zonal flows are regulated by tertiary instability.

Tertiary Instability Threshold Sensitive to Safety Factor

- Full formula for threshold of tertiary mode remains to be discovered. Numerical tests show magnetic shear and safety factor raise threshold. Example: safety factor.



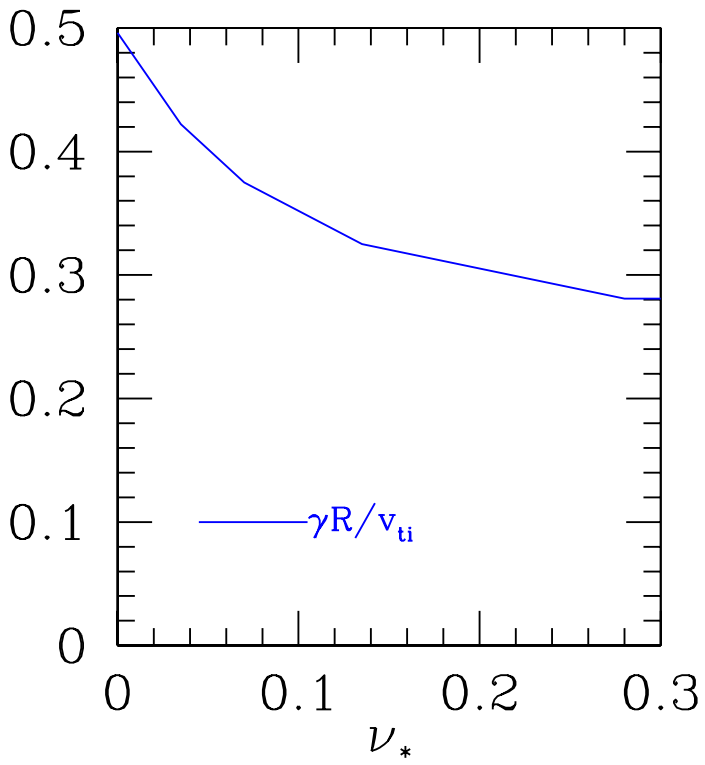
Experimental data consistent with Dimits shift



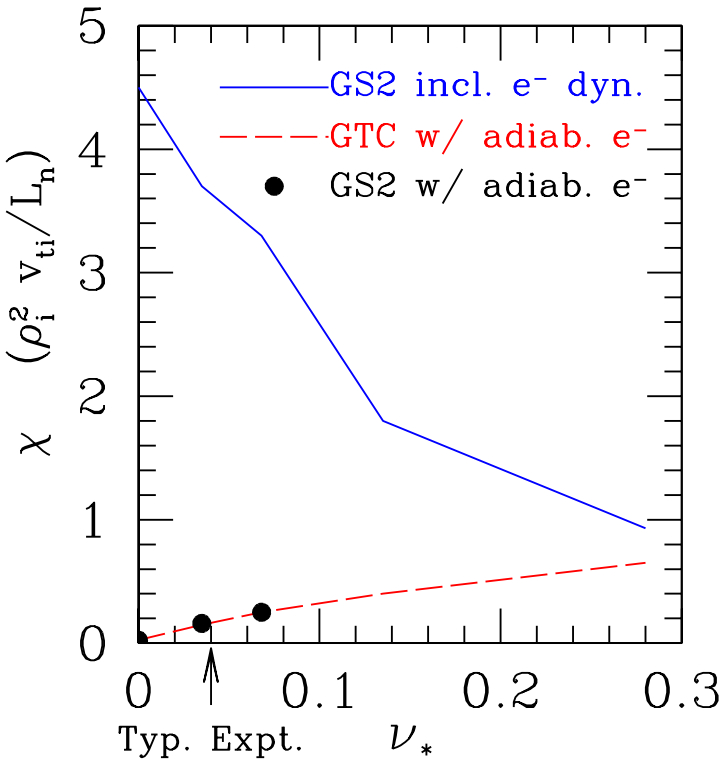
- Comparison with ELMy H-mode from C-Mod; case for which IFS/PPPL models are clearly too high
- Half radius, shot 960116027, $t = 0.9$, $q = 1.3$.
- `GS2` simulations with gyrokinetic electrons and ions, collisionality, general geometry
- Nonlinear upshift of R/L_{Tcrit} clearly improves agreement
- Note: Collisions *increase* upshift and *reduce* transport (because non-adiabatic electron response is reduced) – opposite to finding of Lin, Diamond, *et al.*

Collisional reduction of trapped electron response to ITG turbulence

- Lin simulations assumed adiabatic electron response
- Non-adiabatic electron response due to trapped electrons significant at typical core collisionalities
- Non-adiabatic electrons increase ITG growth rate (well-known, ignored by Lin, *et al.*)



Dominant core collisional effect: ν_{ei}



- Cyclone parameters except $R/L_T = 5.3$ and trapped electrons included.
- Increasing collisionality in experimental range of collisionality
- χ strongly *decreases* – zonal flow damping effect is swamped by decrease in non-adiabatic electron response.

Importance of Dimits Shift for Tokamak Performance

- Dimits shift may be important for understanding cases for which core ion temperature profiles are experimentally determined to be stiff, even though the gradient exceeds the linear threshold of the ITG mode. Because of the strong safety factor scaling, this is probably important mainly at high current.

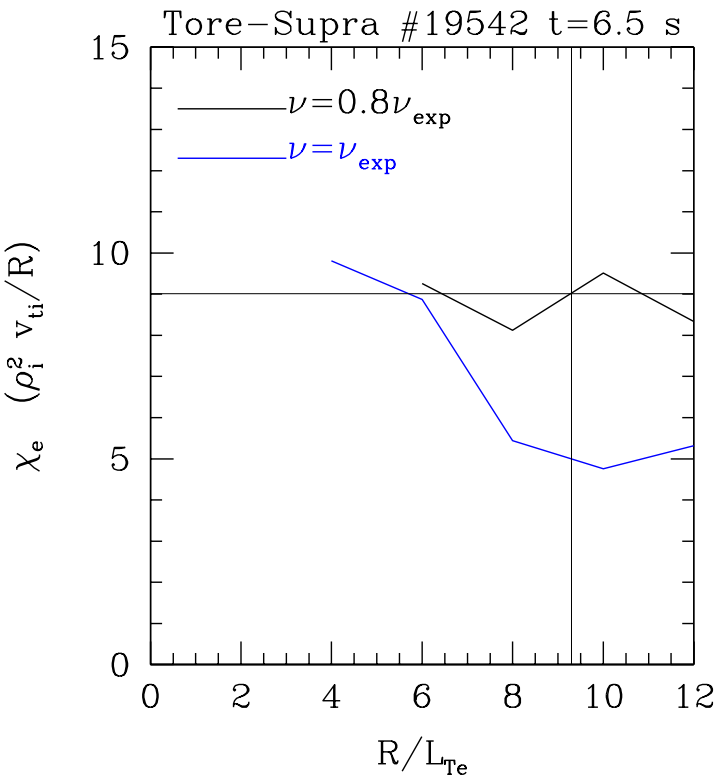
- Known dependences are not consistent with a strong role for stable, self-generated, zonal flows in barrier formation. However, this remains an open subject of substantial interest.

- To the extent that this effect is important, it scales poorly for larger, hotter tokamaks, because of the destabilization associated with non-adiabatic electron dynamics at low collisionality.

- Other questions: Is there an anti-Dimits shift? Is the pedestal width in fact determined by the stability of the pedestal flows to the tertiary instability?

Opportunities for Focus: Electron Energy Transport

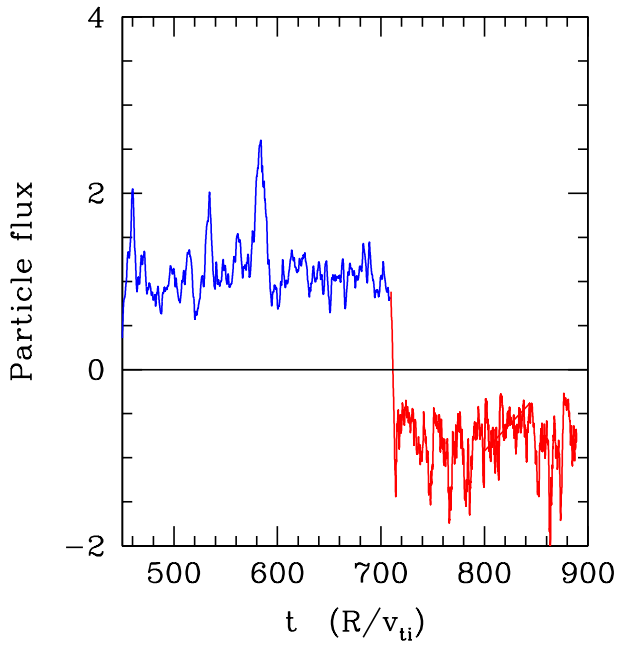
- Experimental tests of transport channels other than ion energy transport needed.
- Tore Supra has excellent data for electron energy transport with which we are comparing. Example:



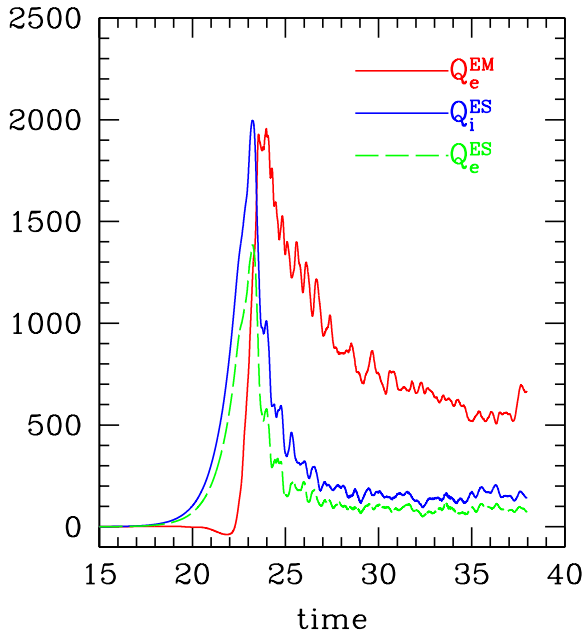
- This is a case for which the ions are cold. Note the unusual behaviour of χ_e *vs.* R/L_{Te}.

Opportunities for Focus: Particle Transport

- Simulations predict an anomalous inward flux of particles – up the gradient. A careful experimental test of this prediction would be interesting.
- Core parameters are taken from Tore Supra. Shown is the turbulent particle flux *vs.* time from a nonlinear simulation.
- At `t=700`, the ion temperature gradient is increased slightly, and the particle flux is observed to reverse, despite the presence of a substantial density gradient.



Opportunities for Focus: Magnetic Transport



- As β approaches the ideal ballooning limit β_c, character of ITG turbulence changes
- Energy transport dominated by

$$Q_e^{EM} \sim \langle q_{\|}\, \delta B_{\perp}/B_0 \rangle$$

- EM particle transport is negligible
- Occurs when finite k_θ component of secondary instability has significant EM component
- Experimentally testable?

Summary

- Gyrokinetic simulations are a new tool with value.
- Progress has been made in understanding which instabilities are important, their relevant threshold criteria, and the dynamics of nonlinear saturation in different scenarios and regimes.
- Stronger ties to an experimental program are needed to make further progress.
- Quirky simulation results may suggest good experimental tests, needed for further acceptance of the basic approach.
- Imperial College is a leader in this area. No other research group has a toroidal gyrokinetic turbulence code that includes electromagnetic perturbations or general geometry, and Imperial is presently at least one year ahead of the nearest competitor in studying non-adiabatic electron dynamics.